Physics AS Module 2 - Mechanics and Molecular Kinetic Theory - Summary Notes

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Thus presented are summary revision notes for the AQA GCE Advanced Level in Physics (Specification A) 5/6451 Unit 2 Mechanics and Molecular Kinetic Theory. They hopefully contain factual information with some background understanding, but few examples - this should mean they are suitable for summary revision. They were originally based off my notes from class which were then checked against the specification for additions to provide something that should be complete.

It is also important to note that physics requires an understanding beyond learning the facts in this document. This must be acquired before these *revision* notes can be of much use.

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1 Physics - the basic skills

Easy marks on exams can be lost due to errors with these mathematical ways of doing science.

1.1 Expressing units

Instead of writing m/s for metres per second, indices notation is used so this becomes ms^{-1}. It is still pronounced in the same way. A more complex example is m/s/s or m/s^2 for acceleration which becomes ms^{-2}. The mathematical rule for indices is

$$ax^{-n} = \frac{a}{x^n}$$

1.2 Expressing answers

Always express answers to three significant figures unless otherwise asked, or use standard form or one of the below power factors, again with three significant figures shown. Never leave answers as fractions, and always give a unit if appropriate. The following must be known for exams.

Name	Symbol	Conversion factor
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

1.2.1 Example of power factor and standard form usage

I have a quantity 640Mg, or 'six hundred and forty mega grams'. I can express it in several ways:

$$640\text{Mg} \times 10^{6} = 640000000\text{g}$$

$$640000000\text{g} \div 10^{-9} = .640\text{ng}$$

$$6.40 \times 10^{8}\text{g to three s.f.}$$

2 Kinematics

2.1 Measuring movement

2.1.1 Distance, displacement, speed and velocity

Distance is a scalar while DISPLACEMENT is a vector, meaning displacement depends on both its magnitude and direction - *displacement, s, is the distance travelled in a given direction.* Speed is the magnitude of velocity.

$$\text{speed (m s}^{-1}) = \frac{\text{distance travelled (m)}}{\text{time taken (s)}}$$

$$\text{average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

VELOCITY can be found using

.08s - average human response time.

$$v = \frac{s}{t}$$

Whilst average velocity can be found using the total displacement and the total time taken, instantaneous velocity can be found by using very small time intervals - where Δt is very small in

$$v = \frac{\Delta s}{\Delta t}$$

Of course, true instantaneous velocity is impossible as the idea of movement requires a change in time and this would not be happening in such an 'instant'.

2.1.2 Acceleration

Acceleration (m s^{-2}) is the rate of change of velocity - $\frac{\Delta v}{\Delta t}$

$$a = \frac{v - u}{t}$$

Because velocity is a vector, the magnitude (speed) doesn't have to be changing - a change of direction (anything not going in a straight line) also has a changing velocity and thus a changing acceleration.

2.2 Showing motion graphically

2.2.1 Displacement-time graphs

Any object's motion can be represented with a graph of displacement vs. time where velocity is the gradient

$$v = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}s}{\mathrm{d}t}$$

The **average velocity** can be found by finding the overall gradient of the graph, and the **instantaneous velocity** at a point can be found by drawing a tangent to the graph at the required value of t, or differentiating

$$\text{instantaneous velocity} = \frac{\Delta s}{\Delta t}$$

A curve in the graph shows non-uniform velocity (acceleration), a flat line means the object is stationary, and a straight line shows a constant velocity - the greater the gradient, the faster the speed.

2.2.2 Velocity-time graphs

A graph of velocity vs. time allows the calculation of acceleration and displacement, as well as representing an object's motion.

The gradient of the graph represents acceleration

$$a = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}v}{\mathrm{d}t}$$

The area under the graph is the total displacement.

A curve in the graph shows non-uniform acceleration, a flat line means constant velocity, and a straight line shows a uniform acceleration - the greater the gradient, the faster the speed. A negative gradient would represent a deceleration or retardation. When asked for this in a question, do not give as a negative acceleration in general.

Negative velocity must be taken into account as velocity and displacement are vectors. The total area between the line of the graph and the x-axis gives the distance travelled and the positive area minus the negative area gives the final journey displacement.

At a-level physics no calculus is required and only $\frac{1}{2}bh$ for a triangle and bh for a rectangle are expected for any calculations.

2.2.3 Acceleration-time graphs

If a body is stationary or moving with a constant velocity then on such a graph $y = 0$ for all values of x. A curve shows non-uniform acceleration and a straight line uniform acceleration.

2.3 Equations of uniformly accelerated motion

These equations apply to any object moving in a straight line (thus) with uniform acceleration. Often referred to as SUVAT given the variables involved.

$$\text{displacement} = s,\ \text{initial velocity} = u,\ \text{final velocity} = v,\ \text{acceleration} = a,\ \text{time} = t$$

- $v = u + at$ for when s is not involved
- $s = \frac{1}{2}(v + u)t$ for when a is not involved
- $s = ut + \frac{1}{2}at^2$ for when v is not involved
- $v^2 = u^2 + 2as$ for when$t$ is not involved.

2.4 Independence of vertical and horizontal motion

Suvat can't be used when objects are not moving in a straight line. However, **horizontal and vertical motion, that is motion at right angles, are totally independent** so we can use suvat and simpler equations on the two components. Often one component has constant velocity and critically t is the same for both.

2.5 Acceleration due to gravity and terminal velocity

Objects in FREE FALL, that is in the absense of resistive forces such as air resistance, accelerate towards the centre of the earth at 9.81m s^{-2} (the force is 9.81N kg^{-1}). As an object falls, air resistance increases as its speed increases, until it matches the force of gravity. The object is then moving at its TERMINAL VELOCITY because it cannot accelerate anymore as the forces are balanced (in equilibrium) and thus there is no resultant.

Objects with a greater surface area have a lower terminal velocity as they create more air resistance so it balances gravity sooner. This may need to be considered in explanation questions in the exam.

2.6 Application - finding g by a graphical method

This is often used as an example on exam questions, so learn it.

The dropping of a steel ball can be used to calculate g, acceleration due to gravity taken as 9.81m s^{-2} on earth at a-level. First, as with all suvat questions, write your data list

$$s = h \text{ (height of fall)},\ u = 0 \text{ ms}^{-1},\ v =?,\ a = g,\ t = \text{time of fall from experiment}$$

$$\text{Using } s = ut + \frac{1}{2}at^2 \text{ we get}$$

$$h = \frac{gt^2}{2}$$

We can fit this to the generic equation for a straight line $y = mx + c$

$$h = \frac{g}{2}t^2 + c$$

$\therefore$ by plotting a graph of h vs. t^2, and gradient will be $\frac{g}{2}$. This allows us to find g by collecting data in the following table:

h (m)	t (ms) (three readings and an average)	$\bar{t}$ (s)	$(\bar{t})^2$ (s^2)

3 Dynamics

3.1 Momentum

Galileo noticed that mass has INERTIA - it resists attempts to change its speed and a force must be applied to cause it to do so. To move faster it must gain momentum and to slow down it has to lose MOMENTUM

$$p = mv$$

Units are $kgms^{-1}$ or Ns as appropriate to the question. Use the former when a direct mass times velocity calculation has been made.

Momentum is a vector with the same direction as the velocity part of the equation.

3.1.1 Conservation of momentum

Momentum is a fundamental quantity in the universe that cannot be created or destroyed. *The total linear momentum of a system is constant provided that there is no resultant force acting.*

$$\sum p_{\text{before}} = \sum p_{\text{after}}$$

3.1.2 Collisions

The above law can be used when two objects collide. Ignoring resistive forces such as friction which remove momentum, the total momentum before the collision must be the same as the total momentum afterwards.

Because the objects are moving they must have kinetic energy

$$E_{\text{k}} = \tfrac{1}{2}mv^2$$

In an ELASTIC COLLISION, the no energy is transferred to the environment and the energy before the collision is the same as after. This is rare however. In an INELASTIC COLLISION, some kinetic energy is lost, commonly in heat due to friction and sound when the objects collide. Momentum, however, is conserved. This allows us to equate momentum even if energy has been lost and calculate velocities before and after collisions. We can also determine whether or not the collision was elastic.

3.2 Newton's Laws of Motion

3.2.1 Newton's first law

"Every object will continue to move with uniform velocity [in a straight line] unless it is acted upon by a resultant, external force."

This comes directly from Galileo's inertia. This law can more clearly be observed in space where gravity and friction are not interfering with continuous movement.

3.2.2 Newton's second law

"The rate of change of an object's linear momentum is directly proportional to the resultant, external force. The change in momentum takes place in the direction of the force."

$$F \propto \frac{\Delta(mv)}{\Delta t}$$

If mass does not change and we measure force in newtons then we can use the corrupted form of the above

$$F = ma$$

This law shows why a car must be stopped slowly in order to create a smaller, and safer force. A lessar deceleration would create a smaller force in the second equation too. IMPULSE is another consequence of this law. It is the idea of creating a large force by increasing the time taken for momentum to change and is applied in sports as 'timing'.

3.2.3 Newton's third law

"If an object, A, exerts a force on a second object, B, then B exerts an equal but opposite force back on A."

This can easily be misinterpreted. The key is to remember that the forces act upon different bodies giving forces in pairs. The forces on any one body do not have to be balanced and equal.

4 Vectors

Scalar quantities only need a magnitude, and a unit, to define them. Vector quantities also need a direction.

4.1 Adding vectors

4.1.1 Graphical 'tip-to-tail' method

By representing vectors with an arrow we can add multiple vectors together. The length of the arrow is proportional to its magnitude while the angle it is placed at to the horizontal shows its direction. Vectors are laid 'tip-to-tail' and the shape is connected up with another line. This line is the resultant vector. If the loop is closed then there is no resultant and the vectors are in equilibrium.

4.1.2 Scale drawing

Using the above method, a scale drawing of the vectors and the resultant can be constructed. Measurements can be taken using a ruler and protractor of the magnitude and direction of the vector. Where the angle is measured from (vertical, horizontal) is important.

Alternatively, the parallelogram method can be used: two vectors are drawn from the same point and then the rest of the parallelogram is constructed. The resultant is the diagonal of the shape. This method may be useful for forces on a moving body.

4.1.3 Geometric method

A combination of trigonometry (including the sine and cosine rules) and Pythagoras' theorem can be used to add vectors together. A sketch following the rules for the above will allow this to be done.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}},\ \cos\theta = \frac{\text{adj}}{\text{hyp}},\ \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 + b^2 = c^2$$

4.2 Components of vectors

Vectors can be split into components at right angles using trigonometry. It is best to learn the following quick trig. rules in order to make calculations faster. For a vector V at angle θ to the horizontal

$$\text{vertical component} = V\sin\theta$$

$$\text{horizontal component} = V\cos\theta$$

If an object is acted upon by multiple forces, RESOLVE each one into its components and then add the vertical and horizontal, considering direction. Then form a triangle from the vertical and horizontal resultants to find the overall resultant.

For a better discussion of this consult mechanics textbooks. This is more of a maths topic than a physics one.

5 Turning moments

Forces can cause objects to rotate and flip about a pivot as well as move. When a force does not act through the centre of mass is may cause rotation. Moments are usually measured in Nm ('newton metres').

The moment, or turning effect, of a force about a point is equal to the force multiplied by the perpendicular distance from the pivot to the line of action of the force.

$$\text{moment} = Fs$$

The PRINCIPLE OF MOMENTS is *for a body in equilibrium, the sum of the anticlockwise moments about any point must be equal to the sum of the clockwise moments about that point.* Additionally for static equilibrium there must be no resultant force on the object causing any movement up or down or left or right.

5.1 The centre of gravity

The centre of gravity of an object is the point about which all the weight appears to act. In a uniform object, this is at the centre - a balanced rod will have a moment created halfway along by the weight, W

$$W = mg$$

5.2 Couples

Every couple has its moment. ∼ D. Duckworth

If two forces of equal magnitude F act an equal distance either side of a pivot in opposite directions they will cause an object to spin. If the total distance between then is d then the moment of the couple is Fd.

6 Work, energy and power

6.1 Work

WORK is done when a force moves through a distance and is measured in joules (J)

$$W = Fs$$

If a force does not cause movement then it does no work. Sometimes the force is not parallel to movement and in such a case only one component of the force, that parallel to the displacement, does all the work.

$$W = Fs\cos\theta$$

6.2 Energy

ENERGY in physics is defined as the ability to do work and is also measured in joules. There are many different equations for energy depending on the situation. Energy is a fundamental quantity that like momentum is always conserved in an isolated system. In fact, *the total energy in an isolated system is constant.*

Kinetic energy and gravitational potential energy can be found with the following

$$\begin{aligned} E_{\text{k}} &= \tfrac{1}{2}mv^2 \\ \Delta E_{\text{p}} &= mg\Delta h \end{aligned}$$

where Δh is the change in vertical height of the object in question. In a situation where an object is losing gravitational potential energy (falling) and thus gaining kinetic energy, by equating the above due to the conservation of energy and ignoring air resistance we can derive an expression for the final speed of the object, v

$$\begin{aligned} \tfrac{1}{2}mv^2 &= mg\Delta h \\ \Rightarrow v^2 &= 2g\Delta h \\ \therefore v &= \sqrt{2g\Delta h} \end{aligned}$$

This is useful in exam questions.

Mass is now seen as a type of energy. If fully converted, this equation gives the amount of energy received. c is the speed of light in a vacuum, $3x10^8\text{ms}^{-1}$

$$E = mc^2$$

6.3 Power

POWER is the rate at which work is done. High power represents lots of work in a short time. Power is measured in joules per second (Js^{-1}) or watts (W)

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

$$P = \frac{\Delta W}{\Delta t}$$

This is often expressed as

$$E = Pt$$

Alternatively by combining $W = Fs$ and $P = \frac{W}{t}$ we can also get the useful

$$P = Fv$$

7 Thermal energy

7.1 Specific heat capacity, c

When different materials are heated to the same temperature, they store different quantities of heat energy. The definition of SPECIFIC HEAT CAPACITY is *the amount of energy needed to raise the temperature of one kilogram of a material through 1K.* It is measured in J kg^{-1} K^{-1}

$$\Delta Q = mc\Delta T$$

7.2 Specific latent heat, l

When a substance is changing state (melting, boiling etc.) then heat energy applied will not cause the temperature of the substance to rise: instead all of the thermal energy will go into changing the substance's state. SPECIFIC LATENT HEAT is defined as *the quantity of heat energy required to change the state of one kilogram of a substance without a change in temperature.* It is measured in J kg^{-1}

$$\Delta Q = ml$$

A different specific latent heat value is needed for each change of state for each substance. However, the specific latent heat of a change is the same as the reverse - for example the specific latent heat of fusion is the same as the s.l.h. of melting.

At a change of state, bonds are broken, particles gain more freedom and move further apart and INTERNAL ENERGY increases. Internal energy is defined as the sum of the potential and kinetic energy of all particles in the substance.

8 The Gas Laws

An IDEAL GAS is one in which molecules are far apart (negligible forces between them) and thus they obey the following laws. There are no collisions, attraction or repulsion between the molecules and they cannot be turned into liquids. Real gases that are hot at low pressure behave much like ideal gases.

The following variables are considered. Temperature must be on the kelvin scale for much of this to work.

pressure = p, volume = V, number of moles = n, number of molecules = N, temperature (K) = T

8.1 Boyle's Law

At constant temperature and mass

$$p \propto \frac{1}{V}$$

This leads to a straight line on a graph of p vs. $\frac{1}{V}$

8.2 The Pressure Law

At constant volume and mass

$$p \propto T$$

This leads to a straight line on a graph of p vs. T when T is measured on the kelvin scale. When measured in degrees Celsius, the line can be extrapolated backwards to the root of the kelvin scale, -273 degrees or the theoretical ABSOLUTE ZERO where all motion stops and gases exert no pressure.

Converting between Celsius and kelvin requires adding or subtracting 273 at a-level. The actual value of absolute zero is -273.15K.

8.3 Charles' Law

At constant pressure and mass

$$V \propto T$$

This leads to a straight line on a graph of V vs. T

8.4 Amount law

At constant volume and temperature

$$p \propto N$$

8.5 Combined gas law

The laws above can be combined, introducing a constant, to

$$pV = nRT$$

or

$$pV = NkT$$

For a constant mass of gas,

$$\frac{pV}{T} = \text{constant}$$

Thus in situations where a gas changes condition, we can use

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

With this equation, different units may be used for pressure and volume as the relationship still exists as long as temperature is still measured in kelvin.

8.5.1 Moles and molar mass

A MOLE is defined as a quantity of gas containing Avogadro's number, N_{A}, of molecules where $N_{\text{A}} = 6.02 \times 10^{23}$.

The MOLAR MASS, M, is the mass of one mole of molecules for a particular gas, e.g. for O_2 the molar mass is 32g mol^{-1}.

$$n\ (\text{mol}) = \frac{m\ (\text{g})}{M\ (\text{g mol}^{-1})}$$

8.5.2 Gas law constants

The Boltzmann constant, k, is the quantity of energy per molecule per degree kelvin and is equal to 1.38×10^{-23}J K^{-1}. Multiplying this by Avogadro's number gives us the amount of energy per mole of gas, R - the molar gas constant. $R = N_{\text{A}} k = 8.31$J $\text{K}^{-1}\text{mol}^{-1}$

9 Kinetic Theory of Ideal Gases

This theory uses Newtonian mechanics to explain pressure and temperature in an ideal gas.

9.1 (Some) assumptions

- Gases consist of a very large number of molecules so statistical methods can be employed
- The molecules are moving rapidly and randomly
- Their motion obeys Newton's laws of motion
- All collisions between molecules and between molecules and the walls of their container are perfectly elastic
- There are no intermolecular forces
- Molecules have negligible volume compared to their container (they are points in space).

Energy is randomly distributed among the particles of a body. When two substances are mixed, the principle of THERMAL EQUILIBRIUM means that areas of greater internal energy will pass on their energy to areas of lower internal energy until the (formerly) greater energy areas have lowered in temperature and areas of less energy have gone up in temperature, reaching an equal temperature across the substance.

9.2 Derivation

9.2.1 Pressure of an ideal gas

Gases exert a PRESSURE on the walls of their containers because their molecules are constantly striking the walls. A gas particle P moving with velocity u moving along the x axis of a container in the shape of a cube chamges momentum as it collides with the wall in a perfectly elastic collision at right angles, and rebounds

$$\begin{aligned} p &= mv \\ \Delta(mv) &= mv - mu \\ &= mu - (-mu) \\ &= 2mu \end{aligned}$$

9.2.2 Time taken to hit the same wall repeatedly

Where l is the length of the box

$$t = \frac{s}{v} = \frac{2l}{u}$$

9.2.3 Rate of change of momentum by Newton's second law

$$\frac{\Delta(mv)}{t} = \frac{2mu}{\left(\frac{2l}{u}\right)} = \frac{mu^2}{l}$$

9.2.4 Force exerted on box wall - Newton$_2$ and Newton$_3$

By Newton's second law, the rate of change of momentum is the force on the box wall the molecule exerts. The box wall exerts an equal and opposite force back on the molecule by Newton's third law.

9.2.5 Total force on one wall of the box

This is the total force of all the N molecules in the box

$$\begin{aligned} \sum F_N &= \frac{m{u_1}^2}{l} + \frac{m{u_2}^2}{l} + \frac{m{u_3}^2}{l} + \ldots + \frac{m{u_N}^2}{l} \\ &= \frac{m}{L}\left({u_1}^2 + {u_2}^2 + {u_3}^2 + \ldots + {u_N}^2\right) \\ &= \frac{mN\overline{u^2}}{l} \end{aligned}$$

Where $\overline{u^2}$ is the mean square velocity.

9.2.6 Pressure on the box wall

$$p = \frac{\text{force}}{\text{area}} = \frac{\left(\frac{mN\overline{u^2}}{l}\right)}{l^2} = \frac{mN\overline{u^2}}{l^3} = \frac{mN\overline{u^2}}{V} \quad \text{(a)}$$

where V is the volume of the box. The total mass of the gas is mN where m is the mass of one particle, and

$$
\begin{aligned}
\text{density}(\rho) &= \frac{\text{mass}}{\text{volume}} \\
\rho &= \frac{mN}{V} \quad \text{(b)} \\
\text{(b)} \rightarrow \text{(a)} \quad p &= \rho\overline{u^2}
\end{aligned}
$$

9.2.7 Accouting for other box faces & directions

We can say that molecules will have components of velocity in all directions, x, y and z. By Pythagoras' theorem we can find the resultant velocity

$$c_1{}^2 = u_1{}^2 + v_1{}^2 + w_1{}^2$$

Taking average velocities accounts for not all molecules travelling with the same speed

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

9.2.8 Average velocities

On average, there will be an equal number of molecules moving in each direction, so

$$
\begin{aligned}
\overline{u^2} &= \overline{v^2} = \overline{w^2} \\
\therefore \overline{c^2} &= 3\overline{u^2} \\
\text{or } \overline{u^2} &= \frac{1}{3}\overline{c^2}
\end{aligned}
$$

9.3 Equations

The full derivation of these equations must be learnt as it could be asked for on the exam.

Recalling $p = \rho\overline{u^2}$ we get

$$p = \tfrac{1}{3}\rho\overline{c^2}$$

and as $\rho = \frac{mN}{V}$

$$
\begin{aligned}
p &= \frac{1}{3}\left(\frac{mN}{V}\right)\overline{c^2} \\
pV &= \tfrac{1}{3}mN\overline{c^2}
\end{aligned}
$$

10 Comparing models for the behaviour of gases

$$pV = NkT \qquad\qquad pV = \tfrac{1}{3}mN\overline{c^2}$$

$$\therefore 3kT = m\overline{c^2}$$

Comparing this to the equation for kinetic energy gives us

$$\overline{E_k} = \tfrac{1}{2}m\overline{c^2} = \tfrac{3}{2}kT = \frac{3RT}{2N_{\text{A}}}$$

for one molecule of gas. For one mole, which is more useful, we get
However,

$$U = \tfrac{3}{2}RT$$

$$\Rightarrow \overline{E_k} \propto T$$

We can thus say that at the same temperature the particles of all gases have the same kinetic energy. However, gas particles move at different speeds because kinetic energy depends on mass as well as velocity.